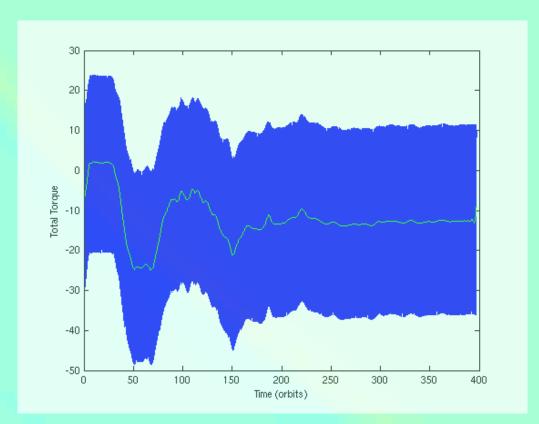
The Eccentric Corotation Torque

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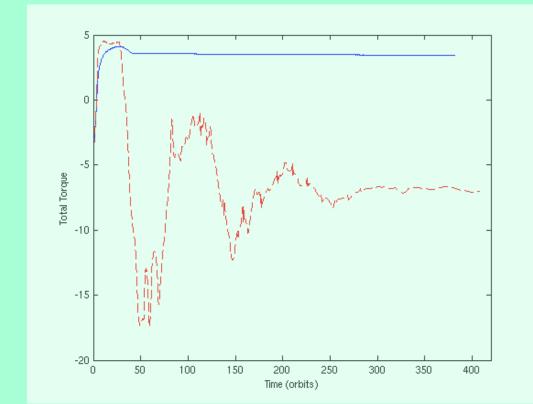
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We run a series of high-resolution 2D hydrodynamic simulations of protoplanets on fixed eccentric orbits embedded in protoplanetary discs. Our protoplanets have a mass equivalent to a 5 Earth-mass planet around a sun-like star.

For a range of disc thicknesses and planetary eccentricities, we simulate inviscid discs with no thermal diffusion (where the corotation torque saturates and leaves only the Lindblad torque) and discs with viscosity and thermal diffusion chosen to optimise the total torque.

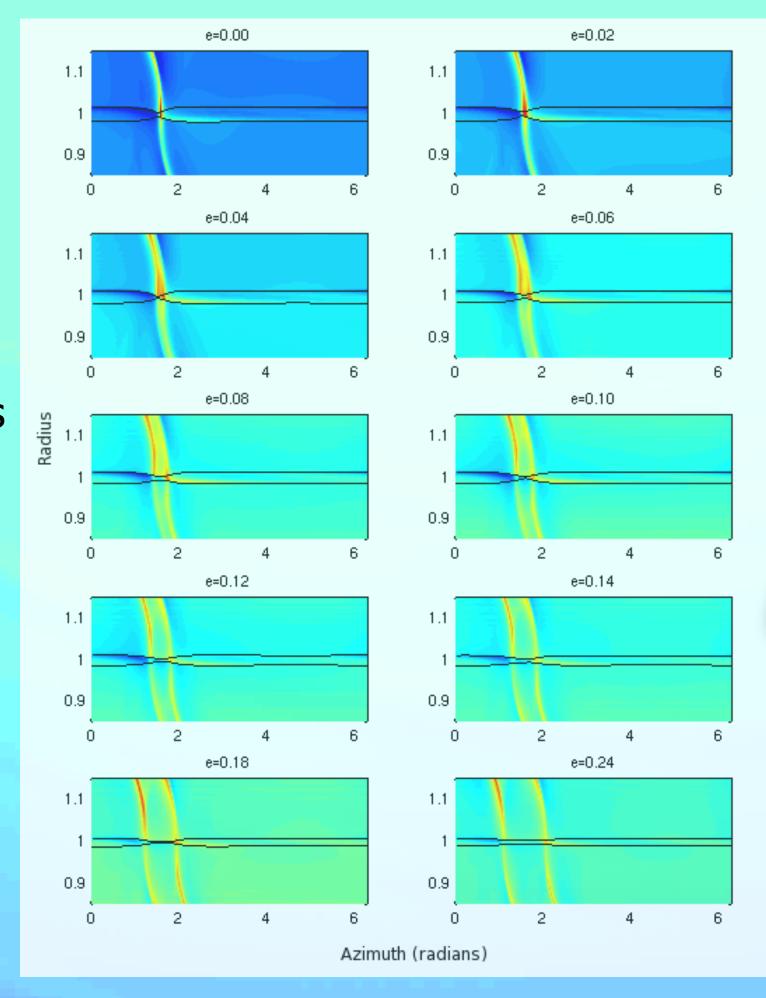


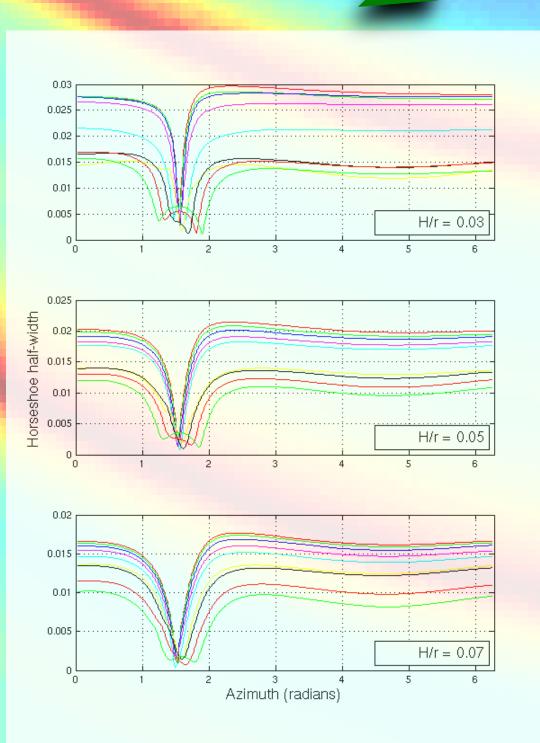
The torque experienced by the planet consists of a slowly varying average value and a rapidly varying contribution from its epicyclic motion. We filter the latter out using a Fourier transform filter.



The filtered torque time-series for two discs-- both with planets on fixed circular orbits in H/r=0.05 discs. In the red case, the corotation torque has saturated. In the blue case, it has been optimised.

By considering timeaveraged density and
velocity fields, we observe
a narrowing of the
horseshoe region-- the
region where disc material
undergoes horseshoe orbits
with respect to the planet.
This material is responsible
for providing the corotation
torque. We suggest that
the narrowing at increased
eccentricity is the cause of
the decreased corotation
torque.



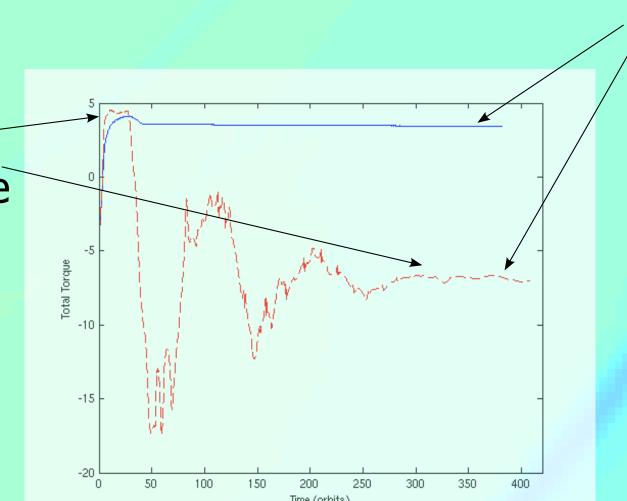


We show the horseshoe half-width as a function of azimuth. Note again the almost monotonic narrowing of the corotation region at increased eccentricity.

In the time-averaged density fields, we also note a splitting of the distinctive spiral density wave into two wake-like structures, due to the planet's velocity relative to the surrounding disc material at apo- and pericentre.

From these simulations, we extract the full non-linear corotation torque using three methods.

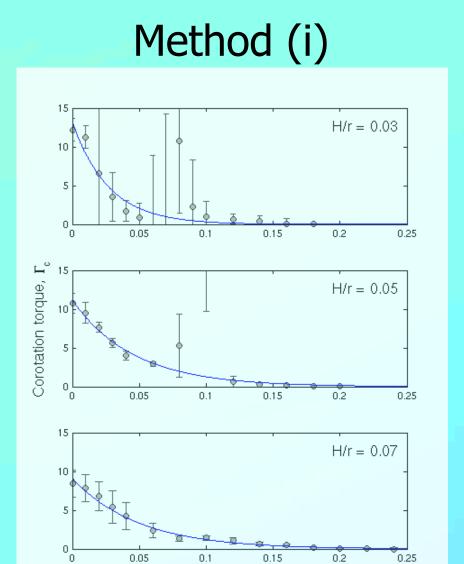
Method (iii): We can use only the simulations where the corotation torque saturates, comparing the initial peak in total toque timeseries to the steadystate value.

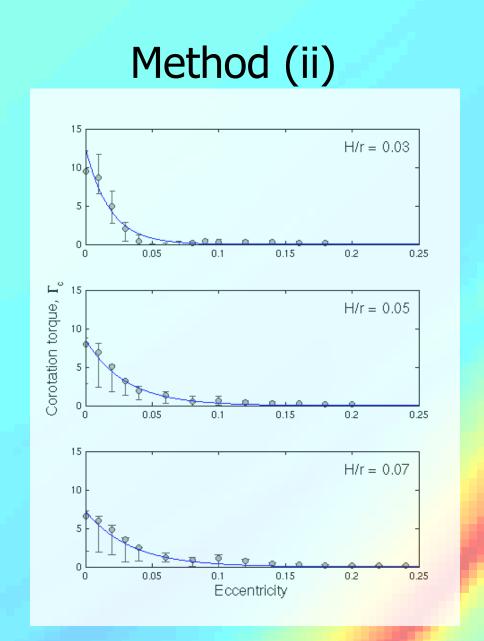


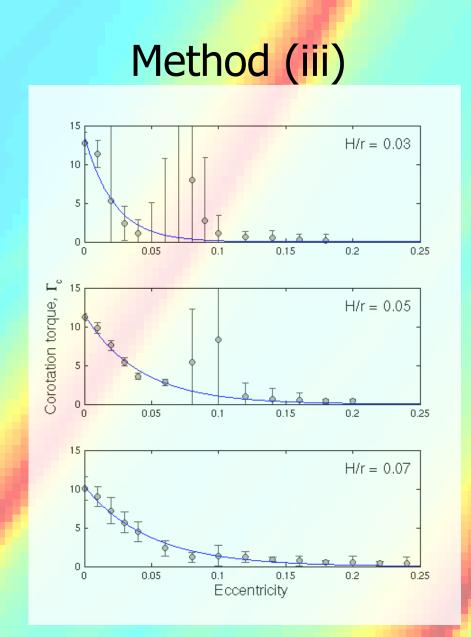
Method (i): We can compare the steady-state total torques for simulations with and without an optimally unsaturated corotation torque.

Method (ii): We can record the torque originating from a defined `corotation region' of the disc in a simulation with a sustained optimally desaturated corotation torque.

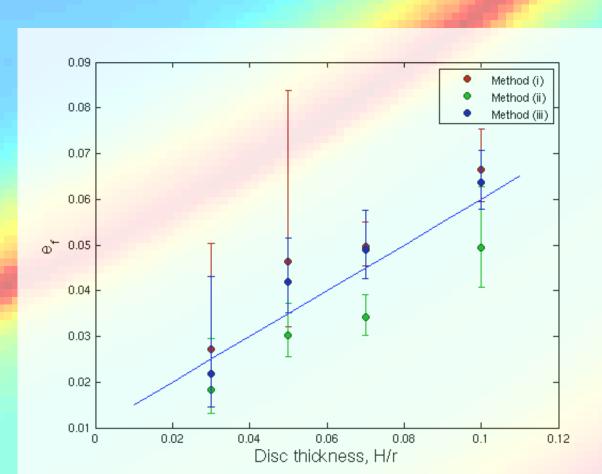
The corotation torque, as measured by the three methods, can be fitted by an exponential attenuation with increasing eccentricity. Some data points do not lie close to the trend lines, particularly in the case of the thinnest discs, as a result of either gap formation beginning to occur or vortices having formed.







We find that the e-folding eccentricity from the above fits goes linearly with disc thickness.



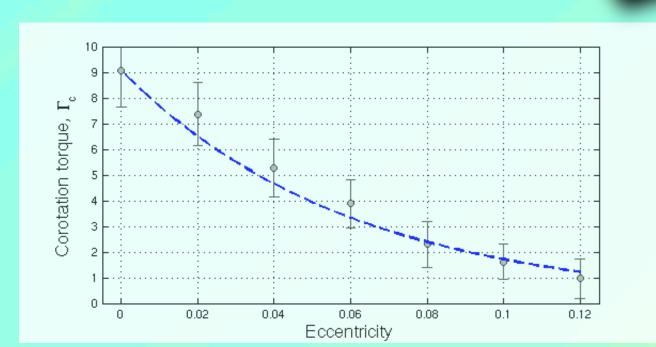
Our empirical fit, relating the corotation torque in the circular case to the corotation torque at a non-zero eccentricity is,

$$\Gamma_e = \Gamma_{circular} \exp(-e/e_f)$$

where

$$e_f = h/2 + 0.01$$

We have tested this relation for a 10 Earth-mass planet and found it to give satisfactory results, suggesting it holds for a range of planet masses.



While previous work has shown that the corotation torque appears to attenuate for eccentric planets, we have run a series of high-resolution simulations and obtained a fitting formula that we have shown to be robust over a range of disc and planet mass parameters. This should provide a useful heuristic, allowing N-body simulations to include a prescription for corotation torque in the case where planetary eccentricities are excited by planet-planet interactions.

Further, locations in the disc where the corotation torque balances the Lindblad torque are known as "zero torque radii" and may act as traps where planetary building blocks can accumulate. Our results suggest that these locations may be a function of eccentricity and that for high enough eccentricities, they may be removed altogether.